Dynamic Pricing for Same-Day Delivery Routing

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Abstract

An increasing number of e-commerce retailers offer same-day delivery. To deliver the ordered goods, providers dynamically dispatch a fleet of vehicles transporting the goods from the warehouse to the customers. In many cases, retailers offer different delivery deadline options, from four-hour delivery up to next-hour delivery. Due to the deadlines, vehicles often only deliver a few orders per trip. The overall number of served orders within the delivery horizon is small and the revenue low. As a result, many companies currently struggle to conduct same-day delivery cost-efficiently. In this paper, we show how dynamic pricing methods are able to substantially increase both revenue as well as the number of customers served the same day. Dynamic pricing incentivizes customers to select delivery deadline options efficiently for the fleet to fulfill. This maintains the fleet’s flexibility to serve more future orders. We model the according dynamic pricing and routing problem as Markov decision process (MDP). To determine suitable state- and option-dependent prices, the state-dependent opportunity costs per customer and option are required. To this end, we develop a guided value function approximation (VFA) approximating the opportunity costs for every state and delivery option with respect to the fleet’s flexibility. As an offline method, the VFA is able to determine suitable prices instantly when a customer orders. In an extensive computational study, we compare the VFA with a policy based on fixed prices and with conventional temporal and geographical pricing policies. The VFA outperforms the benchmark policies significantly leading to both a higher revenue and more customers served the same day.

Keywords: same-day delivery, dynamic vehicle routing, dynamic pricing, stochastic requests, opportunity costs, value function approximation
1 Introduction

Same-day delivery (SDD) is a powerful tool for online retailers to increase sales. SDD is convenient because customers can order online and do not need to go to the store and wait in lines. Further, customers receive their good within a few hours. Thus, SDD narrows the gap of instant-gratification compared to brick and mortar shopping (Anderson 2015). As a result, SDD experiences high two-digit growth rates per year (Yahoo! Finance 2016). Further, the majority of customers is willing to pay delivery fees for SDD (eMarketer 2015). Many retailers offer a set of SDD options differing in delivery speed and price (Grösch 2016). Often, SDD is promised within four-hour delivery deadlines but in some cities like Berlin, Amazon already offers two hour delivery and even partially one-hour express delivery dependent on products and customer locations (Birger 2016, Benedikt et al. 2016).

The combination of SDD and narrow deadlines leads to significant economic challenges for service providers (Ram 2015). Conventional last-mile delivery already causes a majority of the overall delivery costs (Bernau et al. 2016). As Punakivi and Saranen (2001) and Ulmer (2017) show, delivery time commitments additionally increase delivery costs and/or reduce the potential of serving many additional customers and gaining additional revenue. Hence, service providers price different delivery options differently. In their pricing decisions, service providers have two goals in mind. On the one hand, they aim on maximizing the overall obtained delivery fees per day to compensate for the delivery costs. On the other hand, SDD leads to near-instant gratification and may increase the number of orders in the future (Anderson 2015). As a result, the service providers aim on selecting delivery prices leading to both high revenue in delivery fees and a large number of same-day deliveries. Suitable prices may therefore be dynamically adapted with respect to resources available and customer demand as common in many business models like airline ticketing or gasoline retail Borenstein (1996), You (1999). Recently, delivery services draw on pricing mechanisms to control the customer behavior by giving incentives for “efficient” delivery options. This is generally conducted in the field of attended home delivery (Asdemir et al. 2009, Agatz et al. 2011, Yang et al. 2014, Yang and Strauss 2016). These problems differ from SDD because customers are served not the same day. The routing of the delivery vehicles is therefore conducted not simultaneously to the customers ordering but after capturing all orders.
In this research, we transfer the concept of dynamic pricing to same-day delivery. We consider a SDD business model common in e-commerce (e.g., Amazon Prime Now): During the day, customers log on to the retailer’s website and select a set of goods. After finishing the selection, the customer proceeds to the checkout page. At the checkout page, the customers selects the delivery address. Based on the address, the retailer offers a set of different delivery options. These options may comprise SDD with different delivery deadlines (e.g., one-hour, two-hours, four-hours) and a conventional (next-day) delivery. Each option is associated with an according price. Conventional (next-day) delivery is usually free. The customer selects a delivery option based on his or her preference. If a SDD option is selected, the service provider then assigns the order to a vehicle from the delivery fleet to pick up the goods at the warehouse and to deliver the goods to the customer.

The according problem is the dynamic pricing and routing problem for same day delivery (DPPSDD). During a shift, a fleet of vehicles delivers goods from a depot to customers. These customers request orders during the shift and are unknown before the time of their order. For each ordering customer, a set of same-day delivery options, more specific, delivery deadlines is provided. For each delivery option, the provider presents a price. Based on the customer’s (correlated) willingness-to-pay functions, the customer selects a delivery option or rejects the same-day delivery option (and selects conventional delivery). If the customer selects the same-day delivery option, a vehicle picks up the order at the depot and delivers it within the delivery deadline. The objective is to determine a dynamic pricing and routing policy maximizing the expected revenue per shift.

The challenges for the DPPSDD are manifold since it combines stochastic dynamic routing and dynamic pricing. We experience uncertainty in both customer requests and customer choice behavior. Particularly challenging is the instantaneous determination of suitable, state-dependent prices with respect to the choice behavior, vehicle routing, and future customer requests and choices. Prices need to be low enough to encourage customers to use SDD. But prices need to be high enough to generate revenue from the deliveries. Further, individual pricing is a powerful tool incentivizing customers to select a specific option (Agatz et al. 2013). These selections impact the potential of offering same-day delivery to future customers. As an example, an express delivery may result in higher immediate revenue but also in inflexible routing and less future services. A suitable pricing should therefore consider both the customer’s choice and instant revenue and the impact of the fleet’s flexibility to generate future revenues. This impact is quantified in the opportunity costs.
meaning the difference in future revenue in case the customer accepts an option or not. These opportunity costs reflect the expected revenue for the according routing resource consumption. To determine suitable prices and consider both immediate and future revenue, we combine two pricing systems, a (static) basis price system to account for immediate rewards and an individual (dynamic) pricing based on the opportunity costs. If the opportunity costs of an option are low, we offer a static basis price for the delivery option. If they are high, we offer a price with respect to the opportunity costs.

We determine suitable basis prices by policy search based on a restricted class of policies. For the dynamic state-dependent pricing, the opportunity costs for each customer and each option are required. Due to the curses of dimensionality, an exact calculation is not possible. An approximation is necessary. Furthermore, prices need to be derived instantaneously. This prohibits comprehensive online calculations. To approximate the opportunity costs, we present an offline value function approximation (VFA), a method of approximate dynamic programming (ADP, Powell 2011). For each state and option, we approximate the opportunity costs based on a set of state features. These features reflect the fleet’s flexibility to serve future customers in case the option is selected. To achieve a reliable and detailed approximation, we combine non-parametric and parametric VFAs to a meso-parametric VFA (Ulmer and Thomas 2017). We compare the VFA with static pricing and conventional pricing methods based on geography and time. We conduct a computational evaluation for a variety of instances varying in number of orders, customer distribution, customers’ willingness to pay functions, and fleet size. Even though we apply several measures of simplification, the VFA outperforms the benchmark policies significantly with respect to both revenue and number of same-day deliveries per day.

Our contributions are as follows. This work is the first combining dynamic pricing and dynamic vehicle routing. With the DPPSDD, we present a comprehensive Markov decision model for a complex dynamic pricing and vehicle routing problem. For the DPPSDD, we present an anticipatory dynamic pricing and routing policy based on offline ADP. Our policy provides suitable prices instantly and achieves excellent results in comparison to conventional pricing methods. Our work makes two additional contributions to the field of dynamic vehicle routing. We present the first offline ADP-method for a fleet of vehicles in dynamic vehicle routing. Until now, offline ADP-methods were only able to consider dynamic routing of a single vehicle (Meisel 2011, Ulmer et al.
This work further presents the first offline ADP-method for a dynamic vehicle routing problem with temporal commitments, namely, delivery deadlines. The proposed VFA accounts for a state’s flexibility to efficiently serve future requests. The methodology may therefore be transferable to a variety of related problems with deadlines such as food delivery or dial-a-ride.

This paper is organized as follows. In §2, we present related literature from the fields of dynamic vehicle routing and dynamic pricing for delivery services. We model the DPPSDD as a Markov decision process in §3. We discuss the Bellman Equation as a tool for deriving an optimal policy and the challenges to apply the Bellman Equation for the DPPSDD in §4. The dynamic routing and pricing strategies based on opportunity costs are presented in §5. We describe the combination of policy search and VFA in §6. In §7, we present the instance settings and benchmark policies and compare the policies for a variety of instances. The paper concludes with a summary and outlook. Additionally, we offer a comprehensive Appendix where we analyze the VFA and the impact of dynamic pricing in detail.

2 Literature

The research in the fields of pricing and (dynamic) delivery routing is vast. Still, “little research has been carried out at the interface between the two areas” (Yang et al. 2014).

There are only two papers in the literature close to both dynamic delivery routing and delivery pricing. Both papers consider routing and pricing in a different context. Figliozzi et al. (2007) present a dynamic vehicle routing problem, where service providers bid on customer orders in an auction. Instead of a set of prices as in the DPPSDD only a single price per order needs to be determined. Due to the problem’s complexity, they assume a known sequence of orders over time. The only uncertainty is whether the auction for a customer is won. They base their pricing on the opportunity costs defined as the expected loss in revenue by fulfilling the order. We use this procedure in our solution method described in §5. For approximation of the opportunity costs, they use an online one-step lookahead algorithm. Topaloglu and Powell (2007) determine suitable prices per point of time and edge of a given graph for a truck dispatching problem. These prices are determined a priori and are then fixed in the execution of the algorithm where the vehicles dynamically serve transportation requests. Customer choices are modeled implicitly as a demand
function for each edge dependent on the communicated price. While for the DPPSDD, customers selecting an option must be served, in the problem of Topaloglu and Powell (2007), the service provider can select the requests to serve. Further, the travel times per edge is one period meaning that no routes need to be considered in a state. They apply a policy search adapting the prices with respect to the simulation’s outcome and a value-function approximation to decide about the requests to serve.

Beside Figliozzi et al. (2007) and Topaloglu and Powell (2007), problems either address dynamic routing or dynamic pricing in home delivery routing. In the following, we present the relevant literature in these fields. We focus on dynamic decision problems with stochastic requests. These are problems where decisions are made subsequently with respect to newly arriving stochastic customer orders. We first give an overview for the classification. Then, we describe the literature on dynamic routing and delivery routing.

2.1 Overview

We summarize the relevant research in Table 1. Beside Figliozzi et al. (2007) and Topaloglu and Powell (2007), we present research from the field of dynamic routing and delivery routing. We differentiate research with respect to the model and the applied solution method. For the model, we differentiate whether prices are determined explicitly, indicated by a “✓” in the Dynamic Pricing-column, or whether prices are given. Some models like in Campbell and Savelsbergh (2005) allow binary decisions about serving or rejecting the order, often also referred to as dynamic slotting or capacity control. This can be seen as implicit pricing. We indicate this research with “(✓)”. We further depict whether a single vehicle is considered or a fleet in the Fleet-column. In the Dynamic Routing-column, we indicate whether the routing is conducted while customers order as for same-day delivery and not after all orders are collected as for conventional attended home delivery. For some problems like technician routing, customers can be served by a vehicle without a depot return. For the DPPSDD, the vehicles need to pick up the goods at the depot. We indicate models reflecting depot returns with a “✓” Depot Return-column. Models where customers need to be served before a specific deadline are highlighted by a “✓” in the Deadlines-column. A “(✓)” in this column indicates that the vehicles are required to meet a time window (or slot). Some problems explicitly model the customers’ choice behavior. These models are indicated by a “✓” in the Choice
Modeling column. Work where customers choices are the product of other mechanisms like auctions are indicated by “(✓)”. With respect to the applied solution methods, we differentiate whether the method evaluates the decision’s impact to future revenues explicitly (“✓”), implicitly (“(✓)”), or not at all. Methods of explicit evaluation determine the value of a state and decision based on potential future developments. Usually, this evaluation is achieved by methods of ADP. Implicit evaluation is achieved by other measures to incorporate future developments such as the multiple scenario approach by Bent and Van Hentenryck (2004). If the value is determined explicitly, we also indicate when the values are determined, offline a-priori or online in real-time.

2.2 Dynamic Routing

The research on dynamic routing has substantially increased in the last years based on new technologies and business models (Psaraftis et al. 2016, Savelsbergh and Van Woensel 2016). For the same-day delivery problem under consideration, the vehicles need to return to the depot before a customer can be served. Most of the literature focuses on problems where vehicles serve a set of requesting customers over a time horizon without a depot return required. Recently, Azi et al. (2012), Voccia et al. (to appear), Klapp et al. (2016b,a), Ulmer et al. (2016b) present problems where depot returns are required. Azi et al. (2012) and Voccia et al. (to appear) apply the multiple-scenario approach by Bent and Van Hentenryck (2004) for anticipation. Klapp et al. (2016b,a) present a rollout algorithm of an a priori policy to determine the expected values of a decision. The methods by Azi et al. (2012), Voccia et al. (to appear), Klapp et al. (2016b,a) require significant online calculation time and are therefore not applicable for the problem under consideration. The only explicit value determination with deadlines is presented by Ghiani et al. (2009) applying a short term lookahead by sampling future requests. Offline methods presented by Meisel (2011), Ulmer et al. (2017, to appear, 2016b) are generally without the consideration of deadlines or time windows and only consider single-vehicle problem. Pricing in dynamic vehicle routing is only applied with respect to rejecting customers (Meisel 2011, Azi et al. 2012, Ulmer et al. 2017, to appear, 2016b, Klapp et al. 2016a,b). A customer choice model is not considered.
Table 1: Literature Classification

<table>
<thead>
<tr>
<th>Literature</th>
<th>Dynamic Pricing</th>
<th>Fleet</th>
<th>Dynamic Routing</th>
<th>Depot Return</th>
<th>Deadlines</th>
<th>Choice Modeling</th>
<th>Future Value</th>
<th>Offline</th>
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2.3 Attended Home Delivery Routing

Explicit dynamic pricing is only considered in attended home delivery (AHD). For these problems, customers order within a capture phase and the delivery takes place in a subsequent delivery phase. Generally, suitable prices for time windows need to be determined. The customer requests and the pricing decisions are dynamic but the final delivery routing is static. One major difference
between dynamic routing and AHD is the objective. While many dynamic routing problems aim on maximizing the expected number of services given limited routing resources, work on AHD often minimizes the routing costs or route durations serving all requests. The reason is that in AHD, the routing is conducted the next day, and fleet sizes and working hours can be adapted to the realized demands. In dynamic routing, the driver shifts are determined before the requests realize. Because a majority of the transportation costs result from the driver’s wages, routing costs can therefore be viewed as secondary (Thomas 2007). Still, as Ulmer (2016) shows there is a strong connection between minimizing route duration and maximizing the number of customers served.

Early work on AHD is presented by Campbell and Savelsbergh (2005, 2006). Campbell and Savelsbergh (2005) decide about the acceptance of orders with respect to expected future revenues. For this problem, the set of requesting customers is relatively small and request probabilities are known. Campbell and Savelsbergh (2006) present a method to incentive customers’ selection by reducing the price for efficient delivery options. To measure efficiency, Campbell and Savelsbergh (2006) approximate the immediate increase in routing costs.

The work by Asdemir et al. (2009) analytically determines optimal dynamic pricing based on a Multinomial Logit choice model (MNL) with independent customers preferences per product. That means that the individual utility functions of a customer for different options are not correlated. For the DPPSDD, ee assume that a customer has a general tendency whether he or she is willing to pay more or less for same-day delivery. Thus, the utility or willingness-to-pay functions for SDD-option are correlated and the convenient MNL model is not applicable (Gallego and Wang 2014). Instead, our presented method accounts for customer choice modeling by means of simulation. Asdemir et al. (2009) do not consider routing explicitly, but assume a given capacity per ZIP-code. Ulmer and Thomas (2017) consider a dynamic customer acceptance problem with routing and capacity constraints. They present a new and general method of ADP, a meso-parametric VFA. We use this method to determine the opportunity costs of the DPPSDD. Ehmke and Campbell (2014) consider a dynamic problem with stochastic travel times. They decide about customer acceptances with respect to the probability of time window violation. As Asdemir et al. (2009), Cleophas and Ehmke (2014) decide about customer acceptances based on resources available per ZIP-code.

The work by Yang et al. (2014), Yang and Strauss (2016), Klein et al. (2016) determines suitable prices by anticipating future customers. More specific, the prices depend on the expected
opportunity costs. In contrast to the DPPSDD, in Yang et al. (2014), Yang and Strauss (2016), the opportunity costs do not reflect flexibility to serve future orders but the expected delivery costs. To this end, Yang et al. (2014) estimate delivery costs based on a set of historical routes. Yang and Strauss (2016) present a value-function approximation to determine expected delivery costs based on geographical features. Klein et al. (2016) improve the work by Yang et al. (2014) by presenting linear programs to estimate changes in future revenue based on the MNL. As Asdemir et al. (2009), all the presented methods determine prices with respect to an MNL assuming independent choice probabilities.

The DPPSDD is further related to static time slot pricing for attended home delivery (Agatz et al. 2011, Klein et al. to appear, Bühler et al. 2016). In this research area, static prices for time slots and geographical areas are determined. For an excellent review on demand management in attended home delivery, we refer to Klein et al. (to appear).

In our work, we present the first model accounting for dynamic routing and dynamic pricing. We further present the first offline ADP solution method for a dynamic vehicle routing problem with fleets and delivery deadlines.

3 The Dynamic Pricing Problem for Same-Day Delivery

In this section, we define the dynamic pricing problem for same-day delivery (DPPSDD). First, we present a problem statement to introduce the required notation. We then give an example for a decision state. Based on this example, we present a Markov decision process (MDP) model and finally embed the example in the MDP-notation.

3.1 Problem Statement

In the following, we describe the DPPSDD and introduce the according notation. The required notation is listed in Table 2. A fleet of $m$ vehicles $\mathcal{V} = \{v_1, \ldots, v_m\}$ serves a set of dynamically requesting customers $\mathcal{C}$ in service area $\mathcal{A}$ during a shift $T = [0, t_{\text{max}}]$. The point of times and locations of the customer requests follow a spatial-temporal probability distribution. Thus, customers are unknown before their time of request. The loading status of a customer $l(C)$ is initially “not loaded” with $l(C) = 0$. In case the according good is picked up at the depot, the loading status
Table 2: Problem Notation

<table>
<thead>
<tr>
<th>Description</th>
<th>Notation</th>
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<tbody>
<tr>
<td>Service area</td>
<td>$\mathcal{A}$</td>
</tr>
<tr>
<td>Depot</td>
<td>$D \in \mathcal{A}$</td>
</tr>
<tr>
<td>Shift</td>
<td>$T = [0, t_{\max}]$</td>
</tr>
<tr>
<td>Vehicles</td>
<td>$\mathcal{V} = {v_1, \ldots, v_m}$</td>
</tr>
<tr>
<td>Customers</td>
<td>$\mathcal{C}$</td>
</tr>
<tr>
<td>Request time of $C$</td>
<td>$t(C) \in T$</td>
</tr>
<tr>
<td>Loading status of $C$</td>
<td>$l(C) \in {0, 1}$</td>
</tr>
<tr>
<td>Travel time function</td>
<td>$d(C_1, C_2), C_1, C_2 \in \mathcal{C} \cup {D}$</td>
</tr>
<tr>
<td>Delivery options</td>
<td>$\Delta = {\delta^1, \ldots, \delta^o}$</td>
</tr>
<tr>
<td>Next-day delivery option</td>
<td>$\delta^1$</td>
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<tr>
<td>WTP function of $C$ for $\delta^i$</td>
<td>$U^i(C)$</td>
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<td>Expected WTP for $\delta^i$</td>
<td>$u^i$</td>
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<td>Individual preference of $C$ for $\delta^i$</td>
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<td>Current planned route of vehicle $v$</td>
<td>$\theta(V)$</td>
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<td>Current arrival time at customer $C$</td>
<td>$a(C)$</td>
</tr>
<tr>
<td>Service time at customer $C$</td>
<td>$t_s$</td>
</tr>
<tr>
<td>Loading time at depot</td>
<td>$t_D$</td>
</tr>
<tr>
<td>Current planned routes</td>
<td>$\Theta = {\theta(v_1), \ldots, \theta(v_m)}$</td>
</tr>
<tr>
<td>Travel and service duration for route $\theta$</td>
<td>$\bar{d}(\theta)$</td>
</tr>
<tr>
<td>Set of offered deadline options to customer $C$</td>
<td>$\bar{\Delta}(C)$</td>
</tr>
<tr>
<td>Planned route of vehicle $v$ for deadline $\delta^i \in \bar{\Delta}$</td>
<td>$\hat{\theta}^i(V)$</td>
</tr>
<tr>
<td>Planned set of routes for deadline $\delta^i \in \bar{\Delta}$</td>
<td>$\hat{\Theta}^i$</td>
</tr>
<tr>
<td>Price of deadline $\delta^i \in \bar{\Delta}$ for customer $C$</td>
<td>$P^i(C)$</td>
</tr>
<tr>
<td>Selected deadline $\delta^i \in \bar{\Delta}$ for customer $C$</td>
<td>$\delta(C)$</td>
</tr>
</tbody>
</table>

change to “loaded”, $l(C) = 1$. The vehicles deliver goods from the depot $D$ to the customers. The travel durations between customers and/or the depot are determined by function $d(\cdot, \cdot)$. The loading time at the depot is $t_D$. The service time at a customer is $t_s$. The dispatcher maintains and updates a planned route $\theta(V)$ for each vehicle $v$. A planned route $\theta$ is a sequence of $h$ customers $C_i^\theta$ and $p$ depot visits $D_j^\theta$ with according arrival times $a(C_i^\theta)$ ending in the depot.

$$\theta = ((C_1^\theta, a(C_1^\theta)), \ldots, (C_i^\theta, a(C_i^\theta)), (D_1^\theta, a(D_1^\theta)), (C_{i+1}^\theta, a(C_{i+1}^\theta)), \ldots, (C_h^\theta, a(C_h^\theta)), (D_p^\theta, a(D_p^\theta)))$$

The first entry of a route $\theta$ represents the current locations or the next location a vehicle will visit. A route may therefore also start in the depot. Diversion from the current destination is not permitted.
The index $j$ in $D_j$ indicates the depot visits. Index $j = 1$ represents the first planned visit of the depot. Index $j = p$ the last planned visit. The overall set of route plans is $\Theta = (\theta(v_1), \ldots, \theta(v_m))$.

Each customer $C \in C$ requests at a point of time $t(C) \in T$ and is unknown beforehand. An overall set of delivery options $\Delta = \{\delta^1, \ldots, \delta^p\}$ is given. $\Delta$ also contains the conventional, non-SDD option $\delta^1$ (e.g., delivery the next day). For each customer $C$, the provider offers a subset $\bar{\Delta}(C) \subseteq \Delta$ of these delivery options. $\bar{\Delta}$ contains at least the next-day option $\delta^1$. For each offered option $\delta^i \in \bar{\Delta}$, the provider also communicates a price $P^i(C)$. The price for $\delta^1$ is zero: $P^1(C) = 0, \forall C \in C$. Additionally, the provider determines a potential routing $\hat{\Theta}^i = (\hat{\theta}(v_1), \ldots, \hat{\theta}(v_m))$ for each option. The customer $C$ selects an option based on the customer’s willingness-to-pay (WTP). The WTP indicates the highest amount a customer pays for a product (McFadden 1999). More specific, each customer $C$ has a set of individual WTP functions $U^1_1(C), \ldots, U^{\mid\Delta\mid}(C)$ with function

$$U^i(C) = u^i + \epsilon^i(C)$$

for each option $\delta^i$. The expected WTP over all customers for an option $\delta^i$ is $u^i$. For an individual customer, $\epsilon^i(C)$ describes the difference between the WTP of customer $C$ and $u^i$. Generally, the distribution of $\epsilon^i$ over all customers follows a stochastic distribution. For the DPPSDD, we further assume that the individual preferences $\epsilon^i(C)$ per delivery option may be correlated for a customer.

The WTP function for next-day delivery $\delta^1$ is deterministic with $U^1_1(C) = 0$.

Based on the WTP function, the customer selects the option maximizing the utility

$$\delta(C) = \arg\max_{\delta^i \in \Delta} (U^i(C) - P^i(C)).$$

In case all SDD-utilities are negative, the customer selects next-day delivery and is not considered any further. After the selection, the routes are updated and the vehicles proceed with the routing until the next customer orders. The objective is to maximize the expected revenue meaning the sum of prices for the selected options.

### 3.2 Example

Because the MDP model and its notation are complex, we first give an example to describe the functionality of the MDP. Figure 1 shows a decision state on the left and a potential decision on the
right. In the example, the 8th customer requests. The current point of time $t = 30$. The deadline is $t_{\text{max}} = 480$. For the purpose of presentation, we assume a Manhattan-style grid and travel times for each segment of 10 minutes. Loading times at the depot and service times at the customer are 10 minutes as well.

The depot is represented by the black square. White and black circles indicate customers. White circles represent customers not yet loaded on a vehicle. Black circles represent customers with goods already loaded to vehicle. The deadline for each customer is depicted in the adjacent field. Customers 1, 3, 4, and 6 need to be served. Customers 2 and 5 may either have been served already or selected next-day delivery. The new requesting Customer 8 is represented by a white circle with a question mark. For our example, we consider two vehicles indicated by the grey circles. Each vehicle has a planned route represented by the dashed and dotted lines. The first vehicle is on the way to serve Customer 3. The plan then determines a return to the depot and service of Customers 6 and 4. The second vehicle is currently on its way to Customer 1. The current plan determines Vehicle 2 to return to the depot afterward.

Decisions are now made about the prices per deadline and the according routing. To allow the provision of a deadline, an according feasible update of the plan is required. Assuming, the set of delivery options is next-day delivery, 60 minutes delivery, 120 minutes delivery, and 240 minutes delivery. A service within the next 60 minutes is infeasible. All other deadlines can be offered. Hence, 3 potential updates of plans are required. One for each of the two offered same-day deadline as well as one for the case, the customer declines same-day service. These updates may change assignments of not loaded customers as well as the sequences of all vehicles. To keep the example
simple, we assume that the dispatcher maintains the current plans in case the customer declines
the SDD-offer. Further, we only present one potential plan update valid for both offered deadlines
depicted on the right side of Figure 1. The updated plan maintains the plan for Vehicle 1 and assigns
Customer 8 to the second vehicle.

The arrival time of this plan for Customer 8 is 150 minutes and is therefore feasible for both
offered deadlines. For each of these deadlines, a price needs to be determined. Delivery for the next
day is free. The price for 60 minute delivery is n/a. The dispatcher determines prices of 1.5 revenue
units for two-hour delivery and 1.0 revenue units for four-hour delivery. In the stochastic transition,
the customer selects a deadline of 120 minutes based on his WTP function. The realized reward is
1.5. The next decision point occurs, when a new customer requests.

3.3 Route-Based Markov Decision Process Model

In the following, we model the DPPSDD as a Markov decision process. To incorporate the routing
component, we draw on a route-based MDP formulation (Ulmer et al. 2016a). Modeling the
DPPSDD as a route-based MDP enables a direct connection of the model to our solution method
presented later in this paper.

A decision point \( k = 0, \ldots, K \) occurs when a customer requests delivery. The number of
decision points \( K \) is therefore a random variable. A state \( S_k \) in a decision point \( k \) contains the point
of time \( t_k \in T \), the currently planned routes \( \Theta_k = (\theta_k(v_1), \ldots, \theta_k(v_m)) \), the customers including
their deadlines and their loading statuses summarized in

\[
C_k = \{(C^1_k, l(C^1_k), \delta(C^1_k)), \ldots, (C^n_k, l(C^n_k), \delta(C^n_k))\}.
\]

A state further contains the new customer \( C_k \). The overall state definition is therefore

\[
S_k = (t_k, \Theta_k, C_k, C_k).
\]

Decisions \( x_k = (P_k, \Delta_k) \in \mathcal{X}(S_k) \) are made about pricing and according routing updates. The
pricing of decision \( P_k = (P^1_k, \ldots, P^{\Delta_k}_k) \) determines a continuous price \( P^i_k \in \mathbb{R}_+ \) for each delivery
SDD-option \( \delta^i \in \Delta \) for customer \( C_k \). A price of “n/a” reflects that the delivery option is not offered. The set of offered delivery options is therefore \( \bar{\Delta}_k = \{ \delta^i \in \Delta : P^i_k \neq \text{n/a} \} \). For each delivery option
\( \delta^i \), an according update of the routing \( \widehat{\Theta}^i_k \) is determined. For options \( \delta^i \) with price \( P^i_k = \text{n/a} \), the routing update is \( \widehat{\Theta}^i_k = \text{n/a} \) as well. These updates are summarized in \( \Upsilon_k \).

A decision \( x_k \) is feasible, if each of the plan updates \( \widehat{\Theta}^i_k \) in \( \Upsilon_k \) is feasible. An update \( \widehat{\Theta}^i_k \) is feasible if the following conditions hold: The first location and arrival time in the plans are the same as in the original plan for each vehicle meaning no diversion is allowed. It is feasible, if the set of routes contains each customer in \( C_k \) exactly once, the loaded customers are in the same route as before, and unloaded customers are only in sequences after a depot visit. In case of same-day delivery options, the routes contain \( C_k \). Finally, the arrival times reflect travel, service, and loading times and the final arrival time at the depot is before the time limit.

After a decision is selected, a stochastic transition \( \omega_k \) leads to the next decision state. The transition consists of the selection \( \delta(C) = \delta^s \in \Delta \) of customer \( C_k \), and the proceeding with the according routes \( \widehat{\Theta}^s_k \) until a new customer, \( C_{k+1} \) requests at time \( t_{k+1} \). The new state \( S_{k+1} \) contains an update of the routes \( \widehat{\Theta} \) and customers \( C_{k+1} \) as follows:

1. Services: All customers with arrival time \( a(C) + t_s < t_{k+1} \) are removed.
2. Loading: If a route \( \theta \) contains a depot visit with \( a(D) + t_D < t_{k+1} \), this depot visit is removed and all customers in this route are set to loaded.
3. Idling: If a vehicle served all customers in the route, it idles at the depot. They route only contains the depot with arrival time \( a(D) = t_k, \theta = ((D, a(D))) \).

The initial state \( S_0 \) is in \( t_0 = 0 \) with \( C_0 = \emptyset \). The first decision point \( k = 1 \) occurs in \( t_1 = t(C_1) \) when the first customer \( C_1 \) requests. The termination state \( S_K \) is in \( t_K = t_{\text{max}} \). Due to the aforementioned restrictions of the plans, at that point of time, the routes only contain the depot, \( \theta_{K}(v_i) = (D, t_{\text{max}}) \).

A solution for the DPPSDD is a decision policy \( \pi \in \Pi \), a sequence of decision rules \( (X^\pi_0, \ldots, X^\pi_K) \) mapping a state \( S_k \) to decision \( x = X^\pi_k(S_k) \). The objective for the DPPSDD is to find a policy \( \pi^* \) maximizing the expected revenues:

\[
\pi^* = \arg \max_{\pi \in \Pi} \mathbb{E} \left[ \sum_{l=0}^{K} R(S_l, X^\pi_l(S_l)) | S_0 \right].\tag{1}
\]
3.4 Example Revisited

In the following, we revisit the example of §3.2 and embed it in the MDP-notation. The current decision point is $k = 8$ where the 8th customer requests. The current point of time $t_8 = 30$. The set of customers $C_8$ is

$$C_8 = ((C_1, 1, 120), (C_3, 1, 70), (C_4, 0, 250), (C_6, 0, 150)).$$

Mathematically, the plan for Vehicle 1 is

$$\theta_8(v_1) = ((C_3, 60), (D_1, 120), (C_6, 150), (C_4, 180), (D_2, 230)).$$

The plan formulation for the second vehicle is $\theta_8(v_2) = ((C_1, 50), (D_1, 110))$. The state is

$$S_8 = (t_8, (\theta_8(v_1), \theta_8(v_2)), C_8, C_8).$$

Decisions $x_8 \in \mathcal{X}(S_8)$ are made about the prices per deadline $P_8$ and the according routing $U_8$. The updated plans maintain $\theta_k(v_1)$ and assigns Customer 8 to the second vehicle leading to the following update

$$\hat{\theta}_8^{120}(v_2) = \hat{\theta}_8^{240}(v_2) = ((C_1, 50), (D_1, 110), (C_8, 150), (D_2, 190)).$$

The pricing vector is $P_8 = (P^n(C_8) = 0.0, P^{60}(C_8) = n/a, P^{120}(C_8) = 1.5, P^{240}(C_8) = 1.0)$. The decision is $x_8 = (P_8, U_8)$. The pricing vector is $P_8 = (0, n/a, 1.5, 1.0)$ and the routing update is

$$U_8 = ((\theta_8(v_1), \theta_8(v_2)), (n/a), (\theta_8(v_1), \hat{\theta}_8^{120}(v_2)), (\theta_8(v_1), \hat{\theta}_8^{240}(v_2))).$$

In the stochastic transition $\omega_8$, the customer selects a deadline of 120 minutes based on his WTP function. The realized reward is $R(S_8, x_8) = 1.5$. The next decision point $k + 1$ occurs, when a new customer $C_9$ requests.
4 The Bellman Equation

In this section, we recall the Bellman Equation as a tool to determine an optimal policy. Even though we are not able to solve the MDP to optimality due to the curses of dimensionality, we draw on an approximate Bellman Equation in our method of approximate dynamic programming (ADP). In this section, we further introduce the notation of a value function and of opportunity costs both used in our approximation methods. In the final part of this section, we describe how we use ADP to address the individual curses of dimensionality.

4.1 Value Function and Opportunity Costs

To solve Equation (1) and to determine an optimal policy $\pi^*$, the Bellman Equation can be solved in every state $S_k$ maximizing the sum of expected immediate reward of decision $x$ plus the expected future rewards under condition $S_k$ and $x$:

$$\arg \max_{x \in \mathcal{X}(S_k)} \mathbb{E} R(S_k, x) + \mathbb{E} \left[ \sum_{l=k+1}^{K} R(S_l, X_l^{\pi^*_l}(S_l)) | S_k, x \right].$$

The first term of the Bellman Equation reflects the immediate revenue of a decision. Let $\mathbb{P}(\delta^i, S_k, \mathcal{P}_k)$ be the probability that customer $C_k$ selects option $\delta^i$. The expected immediate revenue is therefore

$$\mathbb{E} R(S_k, x_k) = \sum_{\delta^i \in \Delta(C_k)} \mathbb{P}(\delta^i, S_k, \mathcal{P}_k) \times P(\delta^i).$$

The second term of the Bellman Equation is also called the value $V(S_k, x_k)$ of a state and decision pair. The value also depends on probabilities $\mathbb{P}(\delta^i, S_k, \mathcal{P}_k)$ and the individual values for each option $\delta^i$ and the according routing $V(S_k, \tilde{\Theta}_k^i)$:

$$V(S_k, x_k) = \sum_{\delta^i \in \Delta(C_k)} \mathbb{P}(\delta^i, S_k, \mathcal{P}_k) \times V(S_k, \tilde{\Theta}_k^i).$$

In the following, we briefly define the opportunity costs for the DPPSDD because we draw on them later in our pricing decisions. For the DPPSDD, the opportunity costs reflect the expected change in revenue for the resource consumption of an option or, as Asdemir et al. (2009) describes,
“the value lost by allocating capacity for an order using that option.” The opportunity costs are
directly connected to the values. Given the values of a state $S_k$ and a decision $x_k$, we are able
to calculate the opportunity costs $O_i(S_k, x_k)$ of an option $\delta^i$. The opportunity costs depend on
the current state, the pricing vector $P_k$, and the potential routing updates. For each option $\delta^i$, the
opportunity costs are the value-difference between selecting this option or not. If the option is not
selected, the calculation incorporates the probability of selecting an option and the according price:

$$O_i(S_k, x_k) = \sum_{\delta^j \in \Delta, j \neq i} \mathbb{P}(\delta^j, S_k, P_k) \times (V(S_k, \hat{\Theta}^j) + P_k^j(C)) - V(S_k, \hat{\Theta}^i_k).$$

(2)

4.2 Curses of Dimensionality

The optimal values $V(S_k, \hat{\Theta}^i_k)$, $V(S_k, x_k)$, and therefore the optimal policy $\pi^*$ could be calculated
by backward recursion. For the DPPSDD, this is not possible. We experience multiple challenges
due to the Curses of Dimensionality (Powell 2011). We experience high dimensionality in state,
transition, and decision space. State space and transition space are vast because customers request in
the entire service area. To account for the curses in state and transition space, we apply simulation
and forward programming in form of a value function approximation to approximate the values
$\hat{V}(S_k, \hat{\Theta}^i_k)$. Due to the limited online calculation time, the simulation needs to be conducted offline
and the according values need to be stored. Thus, we represent states by a set of features.

For the DPPSDD, particularly the decision space is tremendous. As for most dynamic routing
problems, the assignment and routing decisions of $\mathcal{U}_k$ are vast. Keeping in mind that the runtime is
highly limited, we apply a dynamic routing and assignment heuristic to determine $\mathcal{U}_k$. For each
option, this heuristic draws on an insertion procedure to integrate the new request in one route plan
and maintains all other route plans. The DPPSDD additionally contains a pricing decision $P_k$ for
each option. Finally, the revenue of a decision $R(S_k, x)$ is unknown until the customer selects an
option. The calculation of the expected revenue of a decision is challenging because the selection
probabilities vary per customer and the individual WTP functions of a customer are correlated. For
the determination of prices, we combine a policy search for basis prices with dynamic pricing based
on the opportunity costs. These opportunity costs depend on both the probability of selecting another
option as well as the interplay between future request behavior and routing and pricing decisions.
Thus, we simplify the opportunity costs and approximate them by means of the aforementioned
4.3 Solving the Approximate Bellman Equation

In summary, we apply the following measures to simplify the Bellman Equation:

1. Routing: We draw on an assignment and routing heuristic from the literature.
2. Pricing: We apply a pricing rule based on basis prices and opportunity costs. This rule selects either the basis price or the opportunity costs.
3. Basis Prices: We apply a policy search to determine suitable basis prices. The opportunity costs for each policy in the set of policies is determined by means of value function approximation.
4. Opportunity Costs: For the problem under consideration, the opportunity costs of an option is the difference in revenue of a customer selecting this option or selecting another option. Thus, the costs also depend on the customer’s choice probabilities and the offered prices for the other options as we show in an example in §A.1. We simplify the opportunity costs for an option by calculating the difference between the customer selecting the option δᵢ or declining all SDD-options by selecting δ₁, next-day delivery. To determine the opportunity costs, we apply VFA.
5. Value Function Approximation: The VFA approximates the individual values and therefore the opportunity costs by means of forward simulation and approximate value iteration based on a set of features.
6. States: The values for the observed states need to be stored. Because the number of potential states is vast, we aggregate states to a set of features. As features, we select the point of time, the free time budget, and a parameter indicating the flexibility of a planned routing. We further calculate the value as the sum of the individual values for each vehicle. These individual values indicate how much revenue this particular vehicle is expected to generate based on its current planned route.

We describe all measures and methods in detail in the following. We first present the assignment and routing heuristic in §5. We then discuss how the prices are derived as a combination of fixed basis prices and opportunity costs. In §6, we present the policy search and a meso-parametric VFA
(M-VFA) to determine the values and therefore the opportunity costs for particular delivery options. M-VFA requires the determination of a set of state features. We motivate and define these features and describe how M-VFA uses approximate value iteration to determine the values.

5 Dynamic Pricing and Routing Policy

In this section, we present our dynamic pricing and routing method. We first describe the routing and assignment heuristic to determine the updates of the planned routes for a new customer and a deadline. We then describe the pricing policy based on opportunity costs.

5.1 Routing and Assignment Heuristic

To provide prices immediately, we require a fast assignment and routing heuristic. To determine \( U_k \), we extend the efficient insertion heuristic presented in Azi et al. (2012). For the DPPSDD, we experience tight deadlines meaning that routes usually do only contain a few customers. Thus, we ignore preemptive returns as discussed in Ulmer et al. (2016b). Except in the case the vehicle is currently located at the depot, a route \( \theta \) contains two subtours, divided by a depot visit:

\[
\theta = ((C_1^\theta, a(C_1^\theta)), \ldots, (C_j^\theta, a(C_j^\theta)), (D_1^\theta, a(D_1^\theta)), (C_{j+1}^\theta, a(C_{j+1}^\theta)), \ldots, (C_h^\theta, a(C_h^\theta)), (D_2^\theta, a(D_2^\theta))).
\]

The first subtour contains only loaded customers. The second subtour contains only not loaded customers. Because the order for each new customer needs to be picked up at the depot, customers are only inserted in the second subtour. In the special case that a vehicle is currently located at the depot, the customer can be inserted in the first (and only) subtour.

For a new customer \( C \) and an option \( \delta^i \), the routing heuristic applies the following assignment and routing procedure. In case, a vehicle is currently free and idles at the depot, the routing heuristic selects this vehicle for delivery. If all routes contain customers, the routing heuristic determines the feasible route allowing for the “cheapest” insertion. To this end, the new customer is inserted in the second subtour of the route at the position leading to the smallest extension of travel time. After insertion, the heuristic checks whether all customers of this route are served before their deadline.
The feasible route with the smallest extension is selected and the customer inserted. All other routes remain unaltered. In case no feasible route can be found, the option $\delta^i$ is not offered.

5.2 Pricing Strategy

After determining potential routes for each option, prices need to be derived. The determination of suitable prices is challenging, in particular, because the WTP functions are correlated and the potential of serving future requests depends on the current prices as well as on future pricing and routing decisions.

The probabilities of a customer selecting an option depend not only on the price $P(\delta^i)$, but also on the prices of all other options. The pricing decisions are interdependent. An exact and immediate calculation of the probabilities is therefore challenging. This calculation needs to be conducted in every state if we want to use individual pricing to incentivize certain options. To address this challenge, we implement a pricing rule by combining static basis prices with dynamic opportunity costs. For each option, we only consider two potential prices, a static basis price $p^i_b$ and the opportunity costs $O^i(S_k, \Theta^i_k)$.

We implement basis prices for two reasons. First, many companies offer basis prices as long as resources are available in high volumes (e.g., Uber or Deutsche Bahn). Second, these basis prices can be seen as a policy function approximation guiding the VFA (Powell 2011, p. 242). Without basis prices, the VFA is not able to approximate suitable values as we show in §A.4.3. The pricing rule now offers the basis price for options with low opportunity costs to generate additional revenue. It further sets the price to the opportunity costs for options with high opportunity costs incentivizing customers to select another option. In case, the customer still selects this option, the customer generates the expected revenue of the policy.

Mathematically, we set the price to

$$P^i_k(C) = \max(p^i_b, O^i(S_k, \Theta^i_k)),$$

Figure 2 shows the pricing decision for the example given in §3. The upper portion shows the pricing decision for option 120 minutes delivery deadline. The lower portion represents the pricing calculation for 240 minutes delivery deadline. We assume that basis prices for both deadlines
are given and the expected revenue for selection or not selecting an option are known. The basis prices are represented by the dashed lines. The expected revenues are indicated by the solid lines. We further assume that customers are willing to pay more for a shorter delivery deadline and that offering a shorter delivery deadline leads to less flexibility to generate future revenue. Hence, the basis price and the opportunity costs for option 120 are higher than for option 240 in the example.

We are now able to determine a price for each option. For option 120, the opportunity costs are lower than the basis price. Hence, the option can be provided efficiently and the basis price is offered incentivizing the customer to select this option. For option 240, the opportunity costs are higher than the according basis price. In this case, the higher opportunity costs are offered instead of the basis price. In case, the customer selects option 120, we generate a revenue “higher than expected”. If the customer selects option 240, we generate the exact amount of revenue expected for the according resource consumption.

Even though we assume basis prices and opportunity costs known in the example, their exact calculation for the DPPSDD is challenging due to the curses of dimensionality. In the next section, we describe how we determine suitable basis prices and how we estimate the opportunity costs.
6 Basis Prices and Opportunity Costs

To apply the proposed pricing strategy, suitable basis prices and the according opportunity costs need to be determined. In the following, we describe how we combine policy search with value function approximation. First, we recall policy search. We then present the VFA by describing feature selection and the procedure of the VFA.

6.1 Policy Search

To determine suitable basis prices and estimate opportunity costs, we combine a policy search with value function approximation. A policy search determines the best policy from a set of parameterizable policies by means of simulation. We define policies with varying basis prices. For each policy $\pi_\rho$, the basis prices are a percentage $\rho$ of the expected WTP per option, $p^i_b = \rho \times u^i$. Based on this setting, we apply the VFA as described later in this section for varying $\rho \in \{\rho_1, \ldots, \rho_r\}$. We achieve a policy $\pi^\text{VFA}_\rho$ for each $\rho$.

Eventually, the policy search determines the best percentage parameter $\rho$ from the set of candidates $\{\rho_1, \ldots, \rho_r\}$ by simulating $N$ realizations (days) $\{\omega^1, \ldots, \omega^N\}$. The policy with the highest average revenue is selected. We denote this policy by $\pi^\text{VFA}$:

$$
\pi^\text{VFA} = \arg \max_{\rho \in \{\rho_1, \ldots, \rho_r\}} N^{-1} \sum_{i=1}^{N} \left[ \sum_{k=0}^{K} R(S^i_k, X^\text{VFA}_\rho(S^i_k))|S_0 \right].
$$

The algorithmic procedure of Equation (4) is sketched in Algorithm 1. Given a set of $r$ basis price parameters and a set of $N$ realizations, the algorithm iterates through the basis price parameters to determine the best parameter $\rho^\ast$. Given a parameter $\rho$, the algorithm determines the according policy $\pi^\text{VFA}_\rho$ by function $\text{DeterminePolicy}(\rho)$ drawing on value function approximation based on a set of state features. The according feature selection and VFA-procedure are described in §6.2 and §6.3. The algorithm then evaluates the policy by the $N$ realizations. In case the value of the policy is higher than the currently best policy, the parameter and the according values are stored. Eventually, the algorithm returns the best parameter $\rho^\ast$ and the best policy $\pi^\text{VFA}$.

In the following, we describe how we apply the VFA given a specific $\rho$. First, we describe how we represent states by a set of features.
Algorithm 1: Policy Search

Input: Potential Basis Price Parameters $\rho_1, \ldots, \rho_r$, Realizations $\omega^1, \ldots, \omega^N$

Output: Best Parameter $\rho^*$, Best Policy $\pi^{VFA}$

1 $\rho^* \leftarrow 0$ // Initialization of Best Parameter
2 $V^* \leftarrow 0$ // And Best Value
3 $\pi^{VFA} \leftarrow \text{NaN}$ // And Best Policy
4 for all $\rho \in \{\rho_1, \ldots, \rho_r\}$ // For Each Parameter Candidate
5 do
6 \hspace{1em} $\pi^{VFA}_\rho \leftarrow \text{DeterminePolicy}(\rho)$ // Generate Policy via VFA
7 \hspace{1em} $V \leftarrow 0$
8 \hspace{1em} for all $\omega \in \{\omega^1, \ldots, \omega^N\}$ // Evaluation
9 \hspace{2em} do
10 \hspace{3em} $V \leftarrow V + \frac{1}{N} \text{Value}(\pi^{VFA}_\rho, \omega)$ // Solve Realization
11 \hspace{2em} end
12 \hspace{1em} if $V > V^*$ // If New Highest Value
13 \hspace{2em} then
14 \hspace{3em} $\rho^* \leftarrow \rho$ // Update of Best Parameter
15 \hspace{3em} $V^* \leftarrow V$ // And Best Value
16 \hspace{3em} $\pi^{VFA} \leftarrow \pi^{VFA}_\rho$ // And Best Policy
17 \hspace{2em} end
18 end
19 return $\rho^*$, $\pi^{VFA}$

6.2 Feature Selection

To apply an offline VFA, the approximated values $\hat{V}^i(S_k, \hat{\Theta}_k^i)$ for each state and route update need to be stored. As aforementioned, the state space is vast. To this end, we represent states and route updates by a set of state features. Because the number of features substantially increases with the number of vehicles, we determine the value $\hat{V}^i$ for each vehicle individually. That means that the overall approximate value is

$$\hat{V}(S_k, \hat{\Theta}_k^i) = \sum_{j=1}^{m} \hat{V}(S_k, \hat{\theta}_k^i(v_j)).$$

The values $\hat{V}(S_k, \hat{\theta}_k^i(v_j))$ indicate how much revenue an individual vehicle $v_j$ is expected to generate given state $S_k$ and routing $\hat{\theta}_k^i(v_j)$. For representation, we select three features. From the literature, we select the point of time $t_k$, and the time budget $b$ (Ulmer et al. 2017). The point of time allows a general estimate about future requests and revenue.

The time budget $b$ of a route $\hat{\theta}_k^i(v_j)$ represents the free time available to serve additional
customers. The free time budget gives an estimate of the amount of resources available to serve future customers. It is defined as the difference between time limit \( t_{\text{max}} \) and the arrival time at the depot \( a(D_t^0) \). Recalling the Example in §3.2, the budget for Vehicle 1 is \( 480 - 230 = 250 \) minutes. The budget for Vehicle 2 and route \( \theta_8(v_2) \) is \( 480 - 110 = 370 \) and \( 480 - 190 = 290 \) for route \( \hat{\theta}_8^{120}(v_2) = \hat{\theta}_8^{240}(v_2) \).

For the DPPSDD, we further need to consider the deadlines \( \delta(C) \) in the second subtour of \( \hat{\theta} \). These deadlines impact the flexibility of our vehicle to integrate future requests. Because our routing heuristic does not change the arrival times for the loaded customers, we only consider the not loaded customers, denoted by \( C_n(\hat{\theta}) \). If the arrival times of these customers in the second subtour are close to the deadlines, the potential of integrating new customers in this subtour is low. If the difference is high, we are more flexible and may be able to integrate additional customers in this subtour. As we show in §A.4.1 in the Appendix, a consideration of flexibility is mandatory to allow a suitable VFA.

To evaluate the flexibility, we define the flexibility parameter \( \psi \) of a planned route \( \theta \) as follows:

\[
\psi(\theta) = \begin{cases} 
|C_n(\theta)|^{-1} \sum_{C \in C_n(\theta)} (\delta(C) - a(C)), & \text{if } |C_n(\theta)| > 0 \\
t_{\text{max}} - t_k, & \text{else}
\end{cases}
\]

The case \( |C_n(\theta)| = 0 \) only occurs in an updated plan if the according vehicle is currently located at the depot. In this case, we set \( \psi(\theta) = t_{\text{max}} - t_k \).

The flexibility in the example of §3.2 for Vehicle 1 is \( \psi(\theta_k(v_1)) = 2^{-1}((150 - 150) + (250 - 180)) = 35 \) and the original flexibility for Vehicle 2 is \( \psi(\theta_k(v_2)) = 480 - 30 = 410 \) since the route does not contain any not loaded customers. While point of time and free time budget for both deadline options \( \delta^{120} \) and \( \delta^{240} \) are identical, the flexibility to integrate further orders into the subtour is different. Given \( \delta^{120} \), the flexibility of Vehicle 2 is zero, \( \psi(\hat{\theta}_k^{120}(v_2)) = 2^{-1}(150 - 150) = 0 \). Given \( \delta^{240} \), the flexibility of Vehicle 2 is 120 minutes, \( \psi(\hat{\theta}_k^{240}(v_2)) = 2^{-1}(270 - 150) = 120 \).

### 6.3 Opportunity Costs and Value Function Approximation

Given a parameter \( \rho \), we now describe how we simplify the opportunity costs and approximate them by means of value function approximation.

As Equation (2) showed, the opportunity costs depend on the selection probabilities of all other
options based on the pricing vector $\mathcal{P}$ and the correlated WTP functions of a customer. In §A.1 of
the Appendix, we give a short example highlighting this interdependency between opportunity costs
and pricing vector. The opportunity costs further depend on the future requests and future routing
and pricing decisions. Thus, we are not able to calculate the exact opportunity costs. To estimate
them, we apply two measures: simplification and approximation. First, we simplify the opportunity
costs of an option $\delta^i$ as the difference between the values of the customer selecting this option or
deciding SDD at all:

$$V(S_k, \hat{\Theta}^1_k) - V(S_k, \hat{\Theta}^i_k).$$

The opportunity costs are simplified to the difference of the values of serving the customer with
$\delta^i$ or not at all. This reflects the opportunity costs in many pricing problems where the customer
only accepts or declines the offer, for example, given in Figliozzi et al. (2007). Still, the customer
choices are considered implicitly by the simulation.

The routing procedure presented in §5 maintains all routes except the route of vehicle $v_s$ the
new customer will be assigned to if option $\delta^i$ is selected. We only consider the difference of this
particular vehicle and rewrite Equation (2) as follows:

$$\hat{O}^i(S_k, x_k) = \hat{V}(S_k, \hat{\theta}^i_k(v_s)) - \hat{V}(S_k, \theta^i_k(v_s)).$$

To determine the values $\hat{V}^i$, we apply a meso-parametric VFA as combination of parametric
and non-parametric VFAs. An M-VFA is similar to “conventional” VFAs, except it approximates
values with both parametric and non-parametric methods. A parametric VFA approximates the
value function based on a set of defined features. Based on these features, the parametric VFA
assumes a functional form of the optimal value function (linear, polynomial, etc.) and approximates
the functions coefficients. Usually, parametric VFAs provide a fast and reliable but imprecise
approximation (Bertsimas and Demir 2002). Non-parametric VFAs approximate individual values
for each realized vector of features. These individual values are stored in a lookup table (LT).
Non-parametric VFAs may allow a detailed approximation for some states but on the expense of
a slow approximation process and unreliability for other states (Bertsimas and Demir 2002). The
M-VFA combines the advantages of parametric VFA and non-parametric VFA and alleviates their
shortcomings. To this end, M-VFA simultaneously approximates both a parametric value function $\hat{\tilde{V}}^p$ and non-parametric individual values $\tilde{V}^n$ stored in a LT.

In the following we describe the parametric and non-parametric component of the M-VFA for the DPPSDD and how the M-VFA combines the values. The detailed approximation algorithm is presented in the subsequent section.

As parametric VFA, we first partition the shift to intervals $\tau_1 \ldots \tau_{t_{\max}}$ of minutes: $\tau_1 = [0, 1), \tau_2 = [1, 2), \ldots, \tau_{t_{\max}} = [t_{\max} - 1, t_{\max}]$. For each of these intervals, we assume a linear functional dependency of parameters $b$ and $\psi$. We further add an abscissa. The (time-dependent) value function of the parametric component is then calculated as

$$\hat{\tilde{V}}^p_\tau(t, \theta) = c^b_\tau \times b + c^\psi_\tau \times \psi + c^a_\tau.$$  

The parametric component of the M-VFA approximates the features $c^b_\tau, c^\psi_\tau, c^a_\tau$ for each interval $\tau$. In our case, we use multiple linear regression based on a set of previous observations.

For the DPPSDD, the feature-space is continuous and three-dimensional, one dimension for each feature. One dimension represents the point of time $t$, one dimension the budget $b$, and one dimension the flexibility $\psi$. To reinforce the approximation process, we apply a dynamic LT procedure (DLT) for the non-parametric M-VFA component. The DLT partitions the dimensions to intervals. Each feature vector is then mapped to one of the partitions. Over the approximation process, the DLT decreases the interval length of entries in case they are sufficiently observed and show a significant variance in the observed values. For a detailed description of the DLT, we refer to Ulmer et al. (2017).

The value of the M-VFA is then a convex combination of the individual values with an a-priori defined parameter $\lambda \in [0, 1]$:  

$$\hat{V} = \lambda \times \hat{\tilde{V}}^p + (1 - \lambda) \tilde{V}^n.$$  

In case, the number of observations of an LT-entry is zero, the M-VFA value is the value of the parametric component. M-VFA simultaneously approximates the values in the LT and the coefficients of the parametric component by means of approximate value iteration. M-VFA starts with initial coefficients and values and repeatedly simulates realizations. Within each simulation
run, decisions are determined based on the current values. After each simulation run, the parametric and non-parametric values in the M-VFA are updated resulting in a new policy. As a result, M-VFA approximates the values over the simulation runs. In §A.2 in the Appendix, we describe the algorithmic procedure of M-VFA for the DPPSDD in detail.

7 Computational Evaluation

In this section, we present the details of the computational evaluation. We first describe the test instance settings. We then discuss the tuning of the VFA and the benchmark policies. We finally compare the results of the different policies for the instance settings. The tuning details for the VFA and the benchmark policies are listed in §A.3 in the Appendix. We omit the depiction of online runtimes because they were generally less than one millisecond per decision point.

7.1 Instances

We compare the policies for 54 different instance settings varying in the number of orders, the number of vehicles, the customer distribution, and the variance in the customers’ WTP functions. We assume a time limit of $t_{\text{max}} = 480$ minutes. The service time at a customer and the loading time at the depot are set to $t_s = t_D = 2$ minutes. The deadlines are 60 minutes, 120 minutes, and 240 minutes.

Customer requests follow a Poisson process over time leading to uniformly distributed request times. Requests only occur before $t = 420$. We consider instance settings with expected numbers of orders of 60, 120, and 180. We test the policies for 1, 2, and 3 vehicles. The vehicles travel with a speed of 20km per hour. We assume Euclidean distances in the service area.

We compare two different spatial customer distributions leading to two different service areas. The depot is located in the center of the respective service area. We test the policies for requests following a two-dimensional normal distribution reflecting a common European city layout as for example given in Braunschweig, Germany. The x- and y-coordinates follow independent normal distributions with standard deviation to 2.5km. This leads to a service area where about 99% of the requests occur in a radius of 8km from the service area center. We also apply the policy to a 10km times 10km service area with uniformly distributed customers.
Finally, we define the user WTP function: we assume a concave development of the mean WTP values of \( u^{60} = 2.0 \), \( u^{120} = 1.5 \), and \( u^{240} = 1.0 \). On average, customers are willing to pay twice as much for next-hour delivery compared to 4-hour delivery. We model correlation as follows. We assume that a customer \( C \) willing to pay more for one same-day delivery option is also willing to pay more for another. To this end, we correlate the sign of \( \epsilon^i \) for varying delta. More specific, first, the sign of all \( \epsilon(C) \) is determined, each with probability of 50%. Then, the value for each \( \epsilon^i \) is determined individually from a normal distribution and transferred to the sign if necessary. We analyze instance settings for normal distribution of \( \epsilon^i \) with coefficient of variation (COV) of 0.1, 0.2, 0.3 on the according mean \( u^\delta \). A low COV of 0.1 represents customers with relatively similar WTP functions while a high COV indicates highly varying individual WTP functions.

For each of the 54 instance settings, we evaluate the policies for 1,000 realizations.

### 7.2 Benchmark Policies

We compare our solution method with three benchmark policies. To analyze the impact of anticipation, we compare M-VFA with the myopic equivalent setting all values to zero. As a result, we obtain a set of policies with fixed prices. The best prices are again determined by policy search. We denote the policy \( \pi^{\text{fix}} \).

To analyze the impact of state-dependent pricing, we additionally compare the VFA with two heuristic pricing policies. The first heuristic reflects pricing with respect to the customer’s location. The second heuristic reflects pricing with respect to the time of the customer’s request. Both policies are combination of fixed basis prices and penalty terms for “inconvenient” customers.

The first heuristic policy \( \pi^{\text{geo}} \) aims on saving resources by offering high prices for customers far away from the depot. To this end, we determine the prices for the customers based on their relative travel time from the depot. We set \( d_{\text{max}} \) as reference indicating the maximal possible travel time for a customer. We then set

\[
P^i(C) = \rho^{\text{geo}} \times u^i + f^{\text{geo}} \times \frac{d(C, D)}{d_{\text{max}}}
\]

The price increases linearly with the customer’s distance from the depot. Parameters \( \rho^{\text{geo}} \) and \( f^{\text{geo}} \) are tuned by means of policy search.
The second policy \( \pi_{\text{time}} \) utilizes the observation of Yang et al. (2014) that “the marginal value of a resource decreases over time because the opportunities to use it are fewer”, a common observation in revenue management (Adelman 2007). To this end, \( \pi_{\text{time}} \) offers high prices in the beginning and decreasing prices over the time horizon:

\[
P_{\pi}(C) = \rho_{\text{time}} \times u^i + f_{\text{time}} \times \frac{t_{\max} - t_k}{t_{\max}}
\]

Again, we tune the parameters \( \rho_{\text{time}} \) and \( f_{\text{time}} \) by policy search. The detailed tuning can be found in §A.3 in the Appendix.

### 7.3 Solution Quality

The individual results are shown in §A.6 in the Appendix. Policy \( \pi_{\text{VFA}} \) provides the best solution quality in 47 of the 54 instance settings. In the following, we show the general results. To this end, we calculate the improvement of policies \( \pi \) compared to the fixed-priced policy \( \pi_{\text{fix}} \). The improvement is calculated as

\[
\frac{\pi - \pi_{\text{fix}}}{\pi_{\text{fix}}}
\]

Figure 3 shows the improvement in revenue and number of customers of the policies compared to the fixed-priced policy \( \pi_{\text{fix}} \). On average, policy \( \pi_{\text{VFA}} \) achieves the highest revenue and the largest number of customers served. The average improvement of \( \pi_{\text{VFA}} \) is 13.2\% in revenue and 3.9\% in customers served. This also means that the number of customers ordering conventional next-day routing is lower than for all other policies. Even though we do not consider the policies impact of the conventional next-day routing, this may lead to less workload for the conventional delivery fleet.

Geographical pricing allows an improvement of 7.7\%. The improvement of temporal pricing is marginal with only 0.2\%. The temporal pricing policy \( \pi_{\text{time}} \) is not suitable for same-day delivery routing. In contrast to next-day attended home delivery, we are not able to “save” routing resources because the vehicles are on the road and the resources are consumed anyhow over time. We show this in §A.4.2 in the Appendix. For \( \pi_{\text{geo}} \), the number of served customers is lower compared to the fixed price policy. The geographical pricing policy \( \pi_{\text{geo}} \) improves solution quality by generally offering high prices for customers further away. Hence, the number of same-day deliveries is lower.
Figure 3: Average Improvement Compared to the Fixed-Price Policy

The significant difference of the results for $\pi^{VFA}$ and $\pi^{geo}$ further indicates that static pricing solely based on geographical districts may not be suitable for highly dynamic SDD. We confirm these assumptions in §A.4.2 in the Appendix. Policy $\pi^{VFA}$ is able to offer reasonable individual prices for some customers distant from the depot, if they can be served efficiently. The number of served customers is significantly higher compared to all other policies. We analyze the reasons in §A.4.3 in the Appendix.

As the individual results in the Appendix show, the improvement of $\pi^{VFA}$ differs slightly with respect to the customer distribution and the COV. On average, the improvement is 15.0% for instances with normally distributed requests and 10.7% for uniformly distributions. This confirms that anticipation for uniformly distributed customers is generally less reliable (Ulmer et al. to appear). The same behavior can be observed for varying COVs in the customers’ WTP functions. For a COV of 0.1, the improvement is 15.8%, for a COV of 0.2, the improvement is 13.3% and 11.9% for a COV of 0.3. Again, a higher uncertainty in the customers’ choices leads to less reliable anticipation and slightly lower improvements.

While the results for varying customer distributions and COVs remain relatively stable, we
experience significant differences in improvement for varying numbers of orders and vehicles as presented in detail in §A.5. Notably, the number of orders differs from the number of realized same-day deliveries because customers may choose regular next-day delivery.

Generally, the solution quality depends not on the overall number of orders or vehicles, but on the number of orders per vehicle as we show in Figure 4. On the x-axis, the expected number of orders per vehicle is shown. On the y-axis the improvement is depicted. The diamonds represent instance settings with 1 vehicle, the squares instance settings with two vehicles, and the triangles instance settings with 3 vehicles. We can observe that the improvement remains relatively constant for instances with the same expected number of orders per vehicle. As an example, we analyze the improvements for 60 expected orders per vehicle in Figure 4. These are overall 18 different instances with 1 vehicle and 60 expected orders, 2 vehicles and 120 expected orders, or 3 vehicles and 180 expected orders. All improvements accumulate in a range between 6.5% and 15.9%. Hence, the number of orders per vehicle is a suitable indicator for the success of dynamic pricing based on $\pi^{VFA}$.

This is also reflected if we consider varying numbers of orders per vehicle. We observe a nearly linear increase of improvement with increasing numbers of orders per vehicles. This can be
explained by the potential of anticipation and the availability of resources. If the number of orders per vehicle is low, anticipation is generally challenging since single orders impact the solution quality significantly. Once the number of orders increases, the impact of single orders decreases and the anticipation of the VFA becomes more reliable. The improvement also increases when the resources available per order decrease. For instance settings with only 20 orders per vehicle, the number of orders per vehicle and hour is less than three. Hence, most of the orders can be served regardless the deadlines and incentivizing customers is not necessary. If the number of orders per vehicle increases, resources per order become increasingly scarce. Anticipation and pricing become important.

8 Conclusion and Outlook

In this research, we have shown the benefit of dynamic pricing for same-day delivery routing. For the considered problem, customers are offered same-day delivery deadline options, each with a price. The challenge is to determine suitable, customer-dependent prices maximizing the current and the expected revenue. This expected revenue depends on the flexibility of the fleet to serve future customers in case an option is selected. To estimate this flexibility, we have developed a combination of policy search and value function approximation to estimate the opportunity costs for an offered delivery option. Based on this VFA, we determine a dynamic pricing rule by means of policy search. All calculations were conducted offline. The online runtime per decision point was generally less than one millisecond. This means that the pricing rule allows instant provision of prices as customers usually expect from an e-retailer. We have compared our policy with conventional pricing policies for a variety of instance settings. Our policy is able to outperform all benchmark policies significantly with respect to obtained revenue. Further, dynamic pricing leads to more customers served the same day.

Future research may focus on extensions of the problem and the methodology. The model may be extended to a multi-depot setting where vehicles may pick up goods on several warehouses distributed in the service area. Further, the model may be extended to allow continuous routing updates also in cases no new customer requested. The impact of the customer’s selection on the regular next-day delivery but also on future customer orders may be integrated. Customer selections
may impact the routing costs of the next day and a customer may increase his or her number of orders in the future if we were able to offer low-priced SDD. Additionally, the SDD routing may be combined with attended home delivery routing. The presented options may then be either same-day delivery deadlines or next-day time windows. Finally, the impact of adding additional “occasional” drivers as discussed in Archetti et al. (2016) to serve customers may be analyzed. In this case, prices need to be offered not only to the customers but also to the drivers.

The applied VFA may be improved by adding additional features. These features may reflect geographical information or information about the fleet, for example, the variance in the vehicles’ flexibility. Finally, in this research, we presented a relatively simple pricing rule. Even this simplified pricing rule leads to substantial improvements. Future research may therefore focus on more sophisticated pricing algorithms able to incorporate correlations, for example, based on nested logit models (Vulcano et al. 2012).

The presented method may finally be transferred to related applications fields with deadlines. One field may be the increasing market of food delivery where vehicles deliver food from restaurants to customers. The VFA may further be adapted to passenger transportation like dial-a-ride where customers are picked up and brought to a location (e.g., a train station) before a deadline. Or the VFA may be applied to service routing such as lockout services where a technician conducts a service at the customer.

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APPENDIX

In the Appendix, we give an example for the interdependency of opportunity costs in §A.1. We present the M-VFA algorithm in §A.2 and describe the tuning of the algorithms in §A.3. We analyze the performance of the policies in detail for one instance setting in §A.4. We further present the results with respect to number of orders and vehicles in §A.5. We finally present the results for the individual instance settings in §A.6.

A.1 Interdependent Opportunity Costs

In the following, we give a simple example to show how the opportunity costs vary with respect to our pricing decisions. Assuming the three feasible options from the example in §3.2 are given, next-day delivery $\delta^n$, 120 minutes deadline $\delta^{120}$, and a deadline of 240 minutes $\delta^{240}$. We show how the opportunity costs for $\delta^{240}$ differ with respect to the price $P^{120}$ of option $\delta^{120}$. To this end, we assume that the optimal routing plans $\hat{\Theta}^n_k, \hat{\Theta}^{120}_k, \hat{\Theta}^{240}_k$ for each option are known. The values of these options are known as well and are $V(S_k, \hat{\Theta}^n_k) = 20, V(S_k, \hat{\Theta}^{120}_k) = 16$, and $V(S_k, \hat{\Theta}^{240}_k) = 18$. For simplification, we assume independent WTP functions, $U^n = 0$, $U^{120}$ is either 2.5 or 5, both with probability of 0.5, and $U^{240} = 2$ is deterministic for all customers.

We now calculate the opportunity costs for $\delta^{240}$ for two different prices $P^{120}$ for $\delta^{120}$. The opportunity costs are the difference between the values of selecting a different option weighted by their probability or selecting $\delta^{240}$.

$$O^i(S, x) = \sum_{\delta^j \in \Delta, j \neq i} \mathbb{P}(\delta^j, S, P) \times (V(S, \hat{\Theta}^j) + P^j(C)) - V(S, \hat{\Theta}^i) \quad (A1)$$

First, we set $P^{120} = 0$ in decision $x^1$. That means that in case $\delta^{240}$ is not selected, the customer always prefers $\delta^{120}$ compared to $\delta^n$. The opportunity costs are

$$O^1(S, x^1) = 1.0 \times (V(S_k, \hat{\Theta}^{120}) + 0) + 0.0 \times V(S_k, \hat{\Theta}^n) - V(S_k, \hat{\Theta}^{240}) = 16 - 18 = -2.$$ 

The opportunity costs are $-2$. We even gain 2 revenue units when the customer selects $\delta^{240}$. That means that we should incentivize the customer to select option $\delta^{240}$ and we may offer a low price.
Even a price of 0 is advantageous.

Now, we set $p_{120} = 4$ in decision $x^2$. That means that in case $\delta_{240}$ is not selected, the customer selects $\delta_{120}$ and $\delta_n$ both with probability 0.5. The opportunity costs are

$$O^i(S, x^2) = 0.5 \times \left( (V(S_k, \hat{\Theta}_{120}) + 4) + V(S_k, \hat{\Theta}_n) \right) - V(S_k, \hat{\Theta}_{240}) = 0.5 \times (20 + 20) - 18 = 2.$$  

The opportunity costs are 2. That means that we lose an expected revenue of 2 if the customer selects $\delta_{240}$ and our offered price should be at least 2.

As we see, by varying the price of one option, we change the opportunity costs for another. Assuming that we use the opportunity costs in our determination of a price, this pricing decision now changes the opportunity costs of all other options. We experience interdependency between the prices and opportunity costs. Therefore, the calculation of optimal prices is challenging even if we have independent WTP functions and access to the exact state values.

### A.2 M-VFA for DPPSDD: Algorithm

The algorithm of M-VFA is described in Algorithm 2. Input for the algorithm is a set of realizations $\omega^1, \ldots, \omega^N$, the mean WTP values $u^1, \ldots, u^{|\Delta|}$, the basis price parameter $\rho$, the set of options $\Delta$, the initial values $\hat{V}$, and the initial tours $\Theta_0$ for state $S_0$. The algorithm now iterates through the set of realizations. At the beginning of each realization $\omega^j$, the tour $\Theta$ is initialized with $\Theta_0$. Within a realization $\omega^j$, the procedure checks with function $\text{Requests?}(t, \omega^j)$ whether there still exists an order after point of time $t$. If so, the next ordering customer $C$ and the request time $t(C)$ are determined with the functions $\text{NextRequest}(t, \omega^j)$ and $\text{RequestTime}(t, \omega^j)$. Based on the new point of time, the routing is updated with $\text{UpdateRouting}(\Theta, t)$. This function cuts all parts of the routes with arrival time earlier than $t$ except for the final depot visit.

For the new customer, options and prices are determined. The set of prices $\mathcal{P}$ is initially empty. For each potential option, the algorithm now determines the according routing with function $\text{Routing}$ following the heuristic presented in §5. If the heuristic returns a feasible routing $\hat{\Theta}^i$ for this option, the algorithm determines the price. Notably, the next-day delivery option $\delta^1$ always returns a feasible routing, $\hat{\Theta}^1 = \Theta$. For all other options, the altered route $\hat{\theta}^i(v_c)$ is identified.
with function $\text{ChangedRoute}(\hat{\Theta}^i, \Theta)$. For this route, the opportunity costs $O^i$ are calculated as the value difference of $\theta(v_c)$ and $\hat{\theta}^i(v_c)$. The value of each route is determined by function $\text{GenerateValues}(t, \hat{\Theta}, \Omega, \Theta)$. This function draws on the features $t, b, \psi$. Features $b, \psi$ are derived from route $\theta$. The observations $\Omega$ are necessary to determine whether a state was already observed in the LT and whether Equation (5) is applied. The price $P^i$ for option $\delta^i$ is now determined as the maximum of the basis price and the opportunity costs. The price for next-day delivery $P^1$ is always 0 because $O^1 = 0$ and $u^1 = 0$. After the algorithm calculated prices for all options with Equation (3), the selected option is determined by function $\text{Selection}(C, P)$ based on the customer’s WTP function and prices $P$. If a same-day delivery option is selected, the routing is updated and the according revenue $P^s$ and routing $\hat{\theta}^s$ are logged. After each realization, the values and observations are updated. Function $\text{UpdateObservations}(\Omega^j, R)$ determines the value meaning the reward realized after the observation in this realization and adds observation and value to the set of previous observations. Based on the observations and previous values, function $\text{UpdateValues}(\hat{\Theta}, \Omega)$ updates the value functions as aforementioned.

### A.3 Tuning

We determine the set of candidate policies $\pi_{\rho}^{\text{VFA}}$ for the policy search by varying $\rho = 0.1, \ldots, 1.0$.

We tune the VFA for each $\pi_{\rho}^{\text{VFA}}$ as follows. We run 1 million approximation runs. We apply pure exploitation in the approximate Bellman Equation. We set the combination parameter to $\lambda = 0.5$. The coefficients of the parametric component are updated by means of multiple linear regression based on the last 200 observations for a particular time interval $\tau$. The initial coefficients are set to zero. The non-parametric component updates the values based on the dynamic LT procedure. The intervals of the DLT start with interval lengths of 16 minutes and subsequently disaggregates the intervals to 8, 4, 2, and 1 minutes. The disaggregation threshold is set to 3.0.

The benchmark policies are tuned similarly to $\pi^{\text{VFA}}$. We vary the parameters $\rho = 0.1, \ldots, 1.0$ and $f = 0.1, \ldots, 1.0$, run 1,000 evaluation runs, and determine the best policy. For $\pi^{\text{geo}}$, we set $d_{\text{max}} = 24$. For the instances under consideration, this indicates the travel duration to the corner of the (quadratic) service area for uniformly distributed customers. This duration is also suitable for the normal distribution because about 99% of the customers lay in a shorter distance.
Algorithm 2: M-VFA for the DPPSDD

Input: Realizations $\omega^1, \ldots, \omega^N$, Mean WTP Values $u^1, \ldots, u^{N'}$, Basis Price Parameter $\rho$, Options $\Delta$, Initial Values $\hat{V}$, Initial Tours $\Theta_0$

Output: Values $\hat{V}$

1 // Initialization
2 $j \leftarrow 1$
3 $\mathcal{O} \leftarrow \emptyset$ // Observations
4 while $(j \leq N)$ // Simulation of N Realizations
5 do
6   $\mathcal{O}^j \leftarrow \emptyset$, $\Theta \leftarrow \Theta_0$, $t \leftarrow 0$ // Initialization for the next Realization
7   while Requests?($t, \omega^j$) // Simulation of one Realization
8     do
9       $C \leftarrow \text{NextRequest}(t, \omega^j)$ // New Customer
10      $t \leftarrow \text{RequestTime}(t, \omega^j)$ // New Point of Time
11     $\Theta \leftarrow \text{UpdateRouting}(\Theta, t)$ // Update of Routes for new Point of Time
12      $\mathcal{P} \leftarrow \emptyset$ // Set of Prices
13     for all $\delta^i \in \Delta$ // Options
14       do
15         if $i > 1$ // If SDD-Option
16           then
17             $P^i \leftarrow \text{n/a}$
18             $\hat{\Theta}^i \leftarrow \text{Routing}(t, \Theta, \delta^i, C)$ // Routing
19           if Feasible($\hat{\Theta}^i$) then
20             $c \leftarrow \text{ChangedRoute}(\hat{\Theta}^i, \Theta)$
21             $O^i \leftarrow \text{GenerateValues}(t, \hat{V}, \hat{D}, \theta(c)) - \text{GenerateValues}(t, \hat{V}, \mathcal{O}, \hat{\theta}(v_c))$
22             $P^i \leftarrow \text{Max}(O^i, \rho \times u^i)$ // Determine Prices
23         end
24       else $P^i \leftarrow 0$
25       end
26     end
27     $s \leftarrow \text{Selection}(C, \mathcal{P})$ // Customer Selection
28     if $s > 1$ then
29       $\Theta \leftarrow \hat{\Theta}^i$ // Update Routing
30       $R_k \leftarrow R_{k-1} + P^s$ // Add Revenue
31       $\mathcal{O}^j \leftarrow \mathcal{O}^j \cup \{(t, \theta^s)\}$ // Save Observation
32       $\mathcal{R} \leftarrow \mathcal{R} \cup \{R_k\}$ // Save Revenue
33     end
34 end
35 // Update Values
36 $\mathcal{O} \leftarrow \text{UpdateObservations}(\mathcal{O}^j, \mathcal{R})$
37 $\hat{V} \leftarrow \text{UpdateValues}(\hat{V}, \mathcal{O})$
38 $j \leftarrow j + 1$
39 end
40 return $\hat{V}$
A.4 Analysis

In the following, we analyze the results of a specific instance setting in detail. We select the instance with 2 vehicles, 120 orders, normally distributed customers and a coefficient of variation of 0.2. This can be seen as the “average” instance setting. We run 100,000 test runs to get smoother values in the analysis. We first analyze the impact of the flexibility parameter to the VFA and the solution quality obtained. We then compare the results with respect to location and time of the requests to the respectively benchmark policies. We finally depict the dependencies between customers served and revenue earned based on the basis price parameters.

A.4.1 The Flexibility Parameter

In this section, we analyze the functionality and the impact of the flexibility parameter $\psi$. To this end, we first show how the parameter affects the value of a state. Then, we compare the approximation process with and without $\psi$.

Figure A1 shows the value for features $t = 240$, free budget $b = 120$ and varying flexibility parameter $\psi$. Parameter $\psi$ is depicted on the x-axis varying between 0 and 60 minutes. The according
approximated value of \((240, 120, \psi)\) is shown on the y-axis. The values differ significantly for different \(\psi\). As expected, we observe a constant increase of the value up to \(\psi = 20\) and a difference between the values for \(\psi = 0\) and \(\psi = 20\) of more than 3 revenue units. A flexibility of \(\psi = 20\) may allow the integration of additional customers into the subtour under consideration while for \(\psi = 0\) the potential for additional integrations is low. For \(\psi > 20\), the value stays more or less constant. In essence, the parameter \(\psi\) is a suitable indicator for the potential of flexible and efficient future services.

To highlight the importance of parameter \(\psi\), in the following, we compare the approximation process for VFAs with and without \(\psi\). The VFA without \(\psi\) is denoted as \(\pi^0\). This policy draws on the same tuning as the original policy \(\pi^{\text{VFA}}\) but sets \(\psi = 0\) for all states. Figure A2 shows the approximation processes for \(\pi^{\text{VFA}}\) (black line) and \(\pi^0\) (grey line). The x-axis represents the number of approximation runs. The y-axis depicts the solution quality. For the purpose of presentation, we depict the average solution quality over 1,000 approximation runs. The approximation process of \(\pi^{\text{VFA}}\) shows the “typical” VFA-behavior of fast initial improvement, constant increase, and convergence. The approximation process for \(\pi^0\) shows a decrease and eventually leads to a revenue of around 90 per run, less than the policy \(\pi^{\text{fix}}\). The flexibility parameter is therefore highly beneficial.
Figure A3: Selected Prices with Respect to Travel Time from Depot to Customer

and mandatory for successful VFA.

### A.4.2 Geographical and Temporal Price Development

In this section, we analyze the prices selected by the customers based on their location and time of request. We analyze how the selected SDD-prices develop with respect to the customer’s distance from the depot and the point of time the customer requests. To this end, we compare the prices with the prices offered by $\pi_{\text{geo}}$ and $\pi_{\text{time}}$ respectively. To allow a comparison, we normalize the price $P^i(C)$ of option $\delta^i$ by $P^i(C) \times (u^i)^{-1}$ to the percentage of expected WTP.

We first analyze the impact of the travel time from customer to depot to the selected prices shown in Figure A3. We compare the results with the static geographical pricing of policy $\pi_{\text{geo}}$. The x-axis depicts the travel time from depot to customer in minutes. The y-axis depicts the normalized average price a customer selects for SDD. As expected, the price for the best tuning of $\pi_{\text{geo}}$ shows a linear increase with respect to the travel duration. Customers far away from the depot pay significantly more for SDD. The difference for $\pi_{\text{VFA}}$ is less distinct. Again, we observe an increase with respect to the travel time from depot to customer but the prices are relatively moderate. This means that
\(\pi^\text{VFA}\) is able to offer reasonable prices to some distant customers in case, these customers can be efficiently served. It also means that static pricing solely on districts may not be suitable for SDD.

Finally, we analyze the development of selected prices with respect to the point of time the customer requests, depicted in Figure A4. As comparison, we present the results for static temporal pricing of policy \(\pi^\text{time}\). As discussed, policy \(\pi^\text{time}\) decreases prices over time to save resources. In the following, we analyze the price development of \(\pi^\text{VFA}\). We observe a relatively constant pricing over time. Only the first and last hour provide cheaper prices. Initially, the vehicles idles at the depot. Hence, sufficient amounts of resources are available and \(\pi^\text{VFA}\) offers lower prices. Because no customers request for \(t > 420\), policy \(\pi^\text{VFA}\) reduces prices for \(t > 360\) to consume the available resources. Eventually, this leads to the same pricing as a myopic policy because the values and opportunity costs for \(t > 420\) are zero.

### A.4.3 Served Customers versus Earned Revenue

In the following, we analyze the impact of tuning parameter \(\rho\) to solution quality and number of served customers.
Figure A5: Served Customers and Obtained Revenue

Figure A5 depicts the average revenue and the average number of served customers for varying tuning parameter $\rho$. The parameter is depicted on the x-axis. The y-axis shows the average revenue and the average number of customers served customers. For the revenue, we observe a constant increase until $\rho = 0.9$. In the special case of $\rho = 0$, we obtain a revenue of zero. In our VFA, the initial values and therefore opportunity costs are set to zero. As a result, the offered price is zero as well and the values remain zero. An approximation is not possible. This observation indicates the value of guidance of VFA by the basis price. The number of served customers remains relatively constant. We do not observe a significant tradeoff “between increasing demand volume and decreasing revenue per order” as discussed in Agatz et al. (2013). The maximum number of customers is served for $\rho = 0.4$. Interestingly, low prices do not necessarily lead to more customers served. Even though the customers increasingly select SDD-options, these customers impede the potential of serving future customers as we show in the following.

To this end, we compare the percentage of customers offered SDD and the percentage of customers selected SDD over time for the policy $\pi_{VFA}^*$ and $\pi_{fix}^*$. The x-axis of Figure A6 depicts the point of time from $t = 0$ up to the cutoff-time $t = 420$. The y-axis shows the percentage of
customers offered at least one SDD-option and the percentage of customers selected SDD for the two policies. The values of policy $\pi^\text{VFA}$ are depicted in black. The values of $\pi^\text{fix}$ are depicted in grey. For the purpose of presentation, we show the average values per hour. Initially, both policies can offer SDD to nearly all customers. For $\pi^\text{VFA}$, we then observe a slight decrease over time until $t = 360$. In the last hour, the percentage of SDD-offers decreases for two reasons. First, the shift of the vehicles ends in $t_{\text{max}} = 480$ limiting the remaining time to serve customers. Second, as we show later in this analysis, $\pi^\text{VFA}$ converges to a “myopic” behavior in the final hours of orders because the opportunity costs are relatively low. In comparison with $\pi^\text{fix}$, $\pi^\text{VFA}$ is able to offer SDD to more customers. The gap between these two policies increases over time. This can be explained by an increasing “inflexibility” of the fleet given $\pi^\text{fix}$. As we can observe, the number of customers selecting SDD is slightly higher for $\pi^\text{fix}$ in the first hours. Further, these customers are not incentivized to select a deadline efficiently to fulfill. This leads to more customers to serve and more challenging deadlines resulting in less flexibility. As a result, the gap between SDD-selections of $\pi^\text{fix}$ and $\pi^\text{VFA}$ closes in the middle of the day and eventually, $\pi^\text{VFA}$ is able to serve significantly more customers.
A.5 Results with Respect to the Number of Vehicles

In this section, we present the results with respect to the number of vehicles as shown in Figure 3. The y-axis depicts the improvement of $\pi^{VFA}$ compared to $\pi^{fix}$. The global x-axis shows the number of vehicles. The local x-axis depicts the expected number of orders. The highest improvement of 36.6% can be observed for the instance settings with 1 vehicle and 180 expected orders. The lowest improvement of $-1.3\%$ is observed for instance settings with 3 vehicles and only 60 expected orders. Generally, we observe an increasing improvement with the number of orders and a declining improvement with respect to the number of vehicles. Both phenomena can be explained by looking at the improvement per instance setting with respect to expected number of orders per vehicle. As discussed in §7.3, this improvement remains relatively constant regardless the number of vehicles and/or orders.

A.6 Results

In this section, we present the results for the individual instance settings. Table A1 shows the average revenue over 1,000 test runs for the individual instance settings and policies. Table A2
shows the according number of same-day services. Table A3 and Table A4 depict the improvement in revenue and customer services with respect to policy $\pi^{\text{fix}}$.

Table A1: Average Revenues

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Table A2: Same-Day Services

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